Simulation of flows in gasdynamic, vehicular and computer networks: similarities, differences and challenges.

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2025

About this review

- I deliberately included seminal old works with high impact (number of citations) or very recent works showing current trends:
  - to collect foundational definitions;
  - to show evolution;
  - to demonstrate the current state of the art.

About analogies

- Analogies as an important tool for world perception
- Similarity theory (similar equations describe similar processes)
- Hristianovich Sergey Alekseevich:
  - An analogy between free-surface river flows and high speed flows

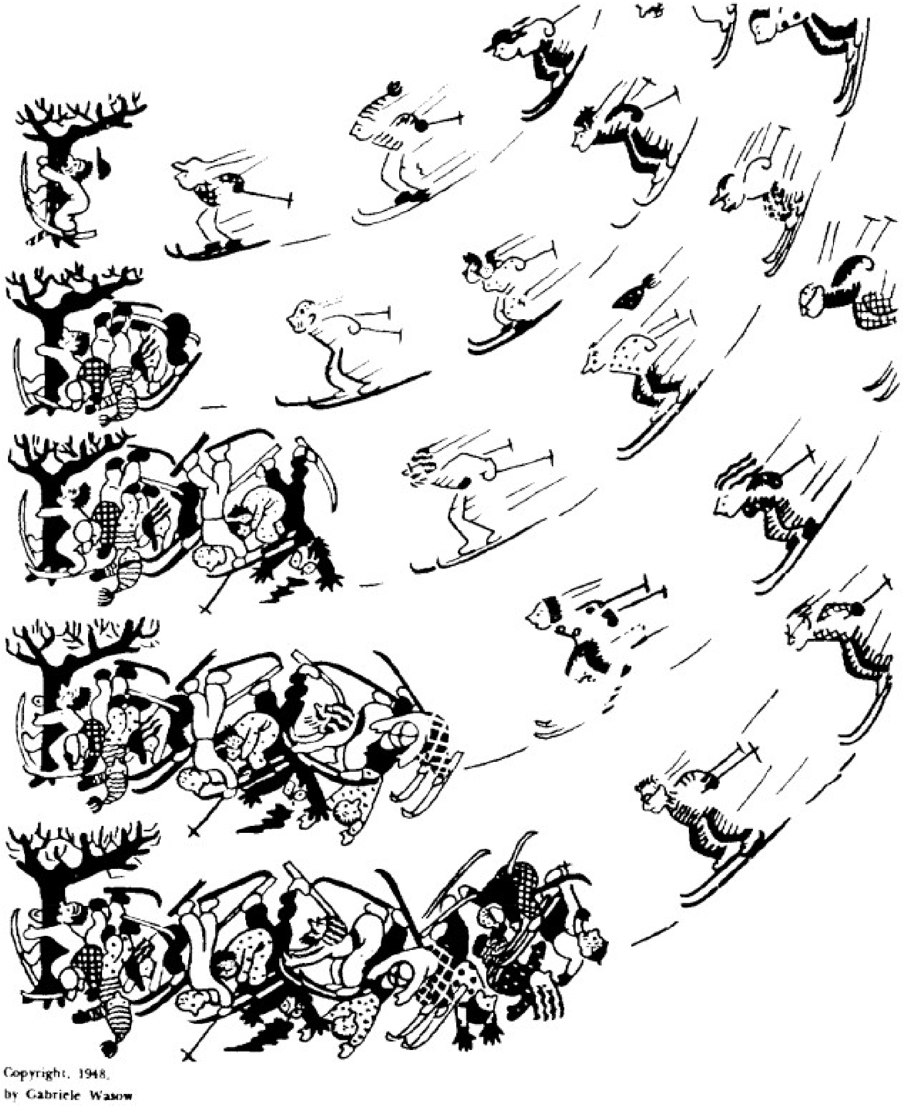
1908 - 2000

Shocks analogies

From [52]:

Explosions, projectiles whizzing by at supersonic speeds, high-speed collisions of solids—what do these phenomena have in common?

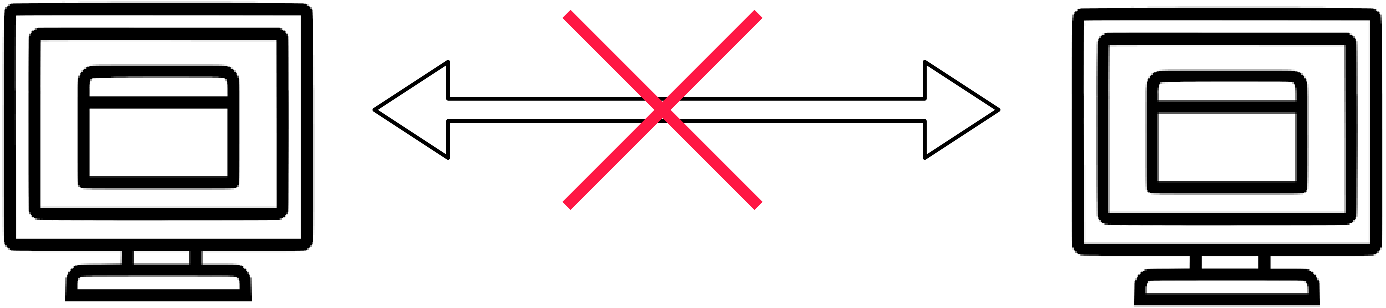
And we add today: water flows during floods, vehicular traffic and data flows in computer networks (probably)



An example of a receding shock wave. From Supersonic Flow and Shock Waves by R. Courant and K. O. Friedrichs (New York:Interscience Publishers, Inc., 1948),

An original problem

- Customers complained about accidental loss of connectivity between their resources or with outer world
- This loss of connectibity was induced by broken ***flows***

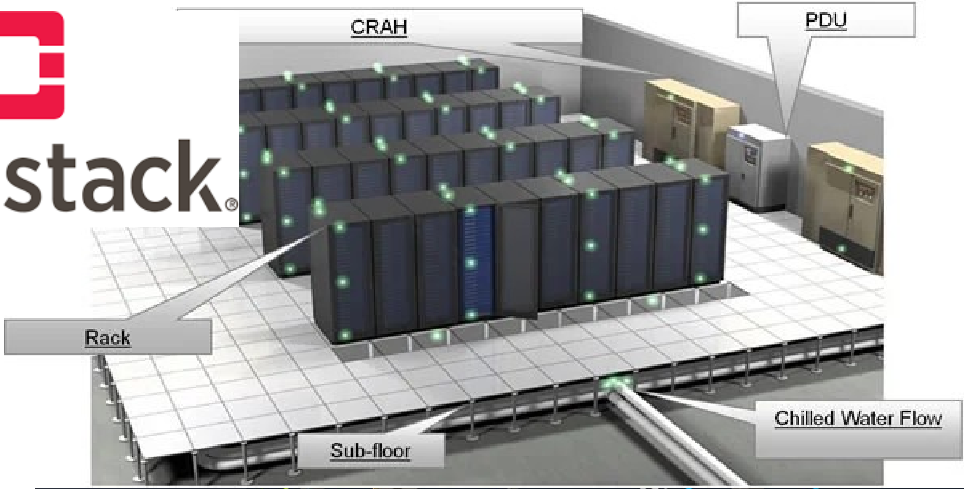


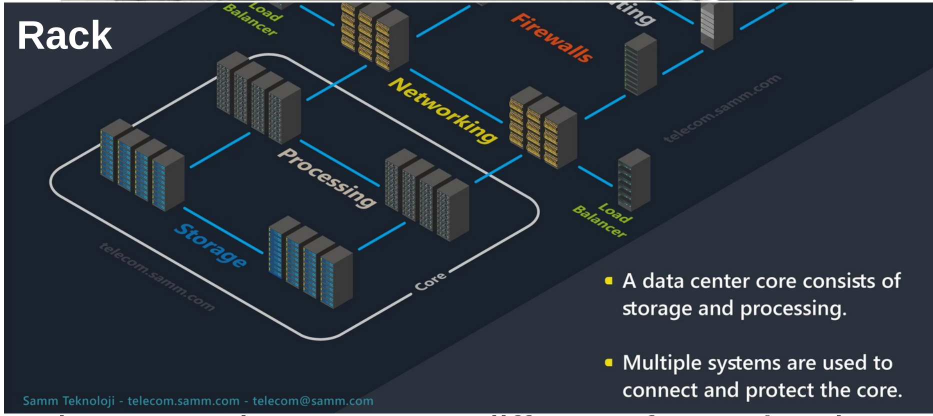
SDN: Virtual computer networks

- A cloud OS: an operating system for distributed computers (data center), it allocates resources
- An SDN: provides connectivity (links) between allocated resources
- Even small errors in some links might lead to serious problem in a whole environment

openstack®

https://mycloudwiki.com/san/data-center-design/





https://telecom.samm.com/what-is-a-data-center-how-are-data-centers-different-from-clouds

Cloud terminology

- A hypervisor: is a physical computer (with CPU, memory, etc) where resources are allocated
- A VM: is a minimum quant of resources to allocate (contains limit for memory, CPU time, etc) for a customer
- A flow: a rule according to which 2 VMs can exchange data

https://www.datacenterknowledge.com/data-center-software/hypervisor-101-understanding-server-virtualization

OpenSDN

- An SDN technology for large distributed computers
- C++(natural language for DES, see appendix)
- Actually plays a role of: a compiler, an OS, a router, a DES simulator
- High complexity (1 500 000 lines of code) → open community to manage it
- Site: https://github.com/OpenSDN-io , don't forget to press "Follow"

OpenSDN community

- Professionals
  - Maintain the code & project
- Companies
  - Run the code
  - Set objectives for development
- Universities (ideally)
  - Grow specialists
  - Solve complicated problems
  - Accumulate knowledge

OpenSDN course at Chulalongkorn university

- In collaboration with NIPA Cloud
- Curriculum: starts on 14th January 2026, 15 weeks
- Instructors: Dr. Abhisak Chulya (Thailand), Dear Jeeraput (Thailand), Tek Charnsilp (Thailand), Moo Pratin (Thailand), Olga Pluzhnikova (Russia), Matvey Kraposhin (Russia)
- Several lessons are dedicated to flow management, because it is a core of all SDN technologies

Definitions of a flow

- Standards-based approach
- Theoretical approach
- Practical approach

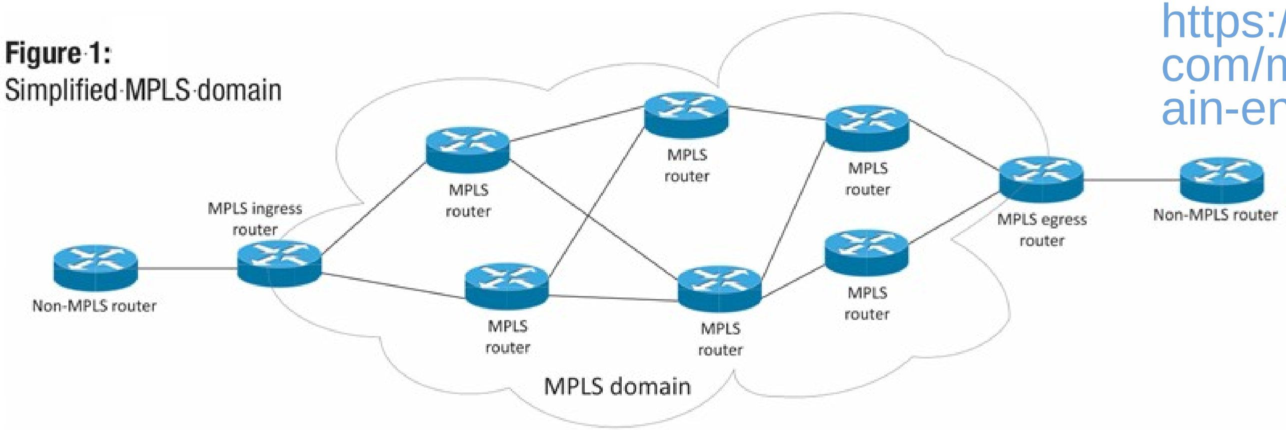
https://mattjhayes.com/2018/09/26/what-is-a-network-traffic-flow/

A standard-based definition

- From RFC 2722, RFC 3697, RFC 3917, RFC 7011:
  - It is a part of data traffic (transmitted data)
  - With common traits
  - Passing through an observation point
  - For a finite period of time
  - Between a source (VM) and a destination (VM)

A formal definition

- According to [3], a flow:
  - is a number (denoted as J, for instance);
  - defined on the edge between a source node and a sink node (consumer)
  - the value of flow is bounded by the throughput of the transport network edge
- If several edges are connected to one node, the sum of incoming fluxes is equal to the sum of outgoing (the divergence theorem)



https://broadbandlibrary.com/mpls-what-is-it-in-plain-english-please/

Applications of flows in computer networks

- When a ***flow*** is
  - a packets count then it is a diagnostic tool
  - a forwarding rule then it is a packets switching tool
  - an edge on a graph then it is a routes representation tool
  - a structure then it is the entry in a database
  - a dynamics of packets in a computer network, it is a theory
- And today it will also mean a flux/stream of data or any other quantity through an edge in a network

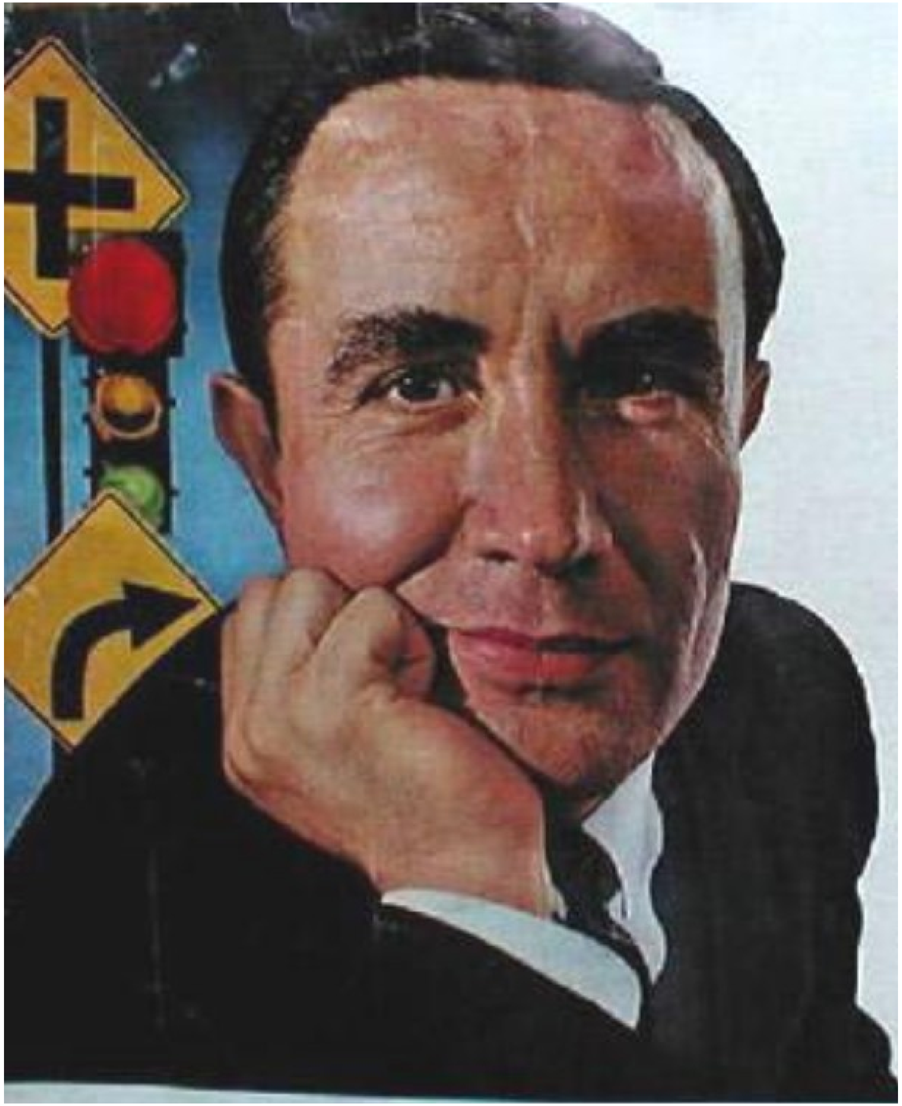
When did the flow-based routing begin?

1937 - 2018

- At the beginning, ***flows*** were used for network diagnostics
- Allegedly, somewhere between 2003:
  - L. G. Roberts, “The Next Generation of IP — Flow Routing,” Proc. SSGRR-2003, Int’l. Conf., L'Aquila Italy, July 29, 2003
- and 2009:
  - [54] Lawrence G. Roberts, A radical new router // IEEE Spectrum, 2009

Historical roots

- Flow networks theory (dates back to 1940-s), but earlier origins can be traced (Kirhoff’s law, divergence theorem, etc) [3]:
  - maximum throughput, shortest path, minimum cost, etc
- Traffic theory   (Traffic flow theory), inc. Lighthill, Witham and Richards macroscopic model
  - congestions, phase transitions, shock waves, etc

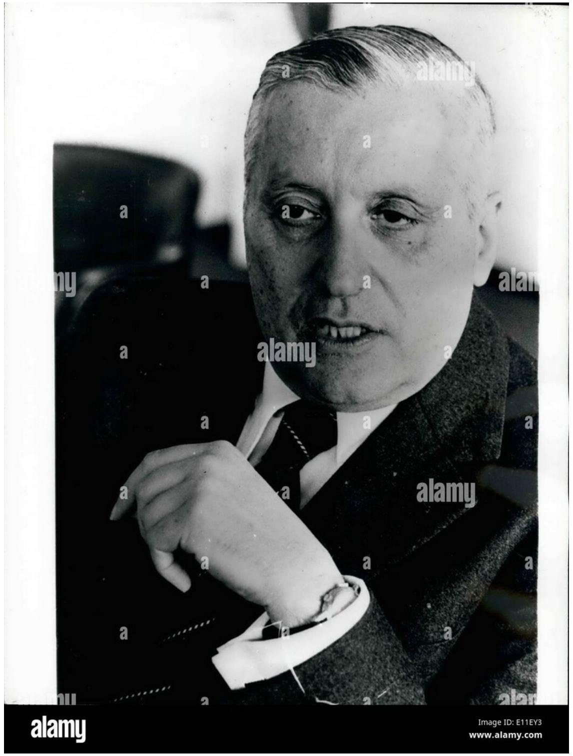


Denos C. Gazis

- The Origins of Traffic Theory [1]
- One of the fathers of modern vehicular traffic theory
- 1930 - 2004

Types of vehicular traffic models

- Microscopic [1]
  - Alexander Reuschel (no photo, unfortunately)
  - Louis A. Pipes (no photo too, unfortunately)
- Macroscopic (Lighthill-Whitham-Richards model)
- Boltzmann equation (Ilya Prigozhine, 1917-2003) for vehicles
  - The Boltzmann-like model filled a lot of gaps in traffic flow modeling [1]

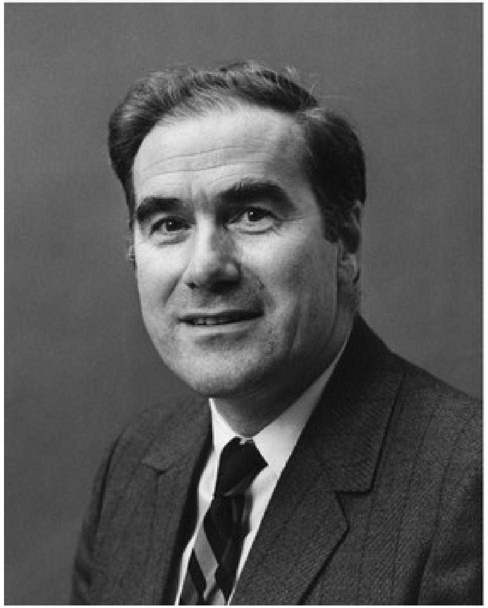


Ilya Prigozhine, https://www.alamy.com/

Lighthill-Whitham-Richards model

James Lighthill, 1924 - 1998
https://www.nationalacademies.org

Lighthill equation,
an AI winter



Gerald B. Whitham
(wikipedia), 1927 - 2014

Whitham's F-function for
far-field supersonic boom prediction
(acoustics due to shock waves)

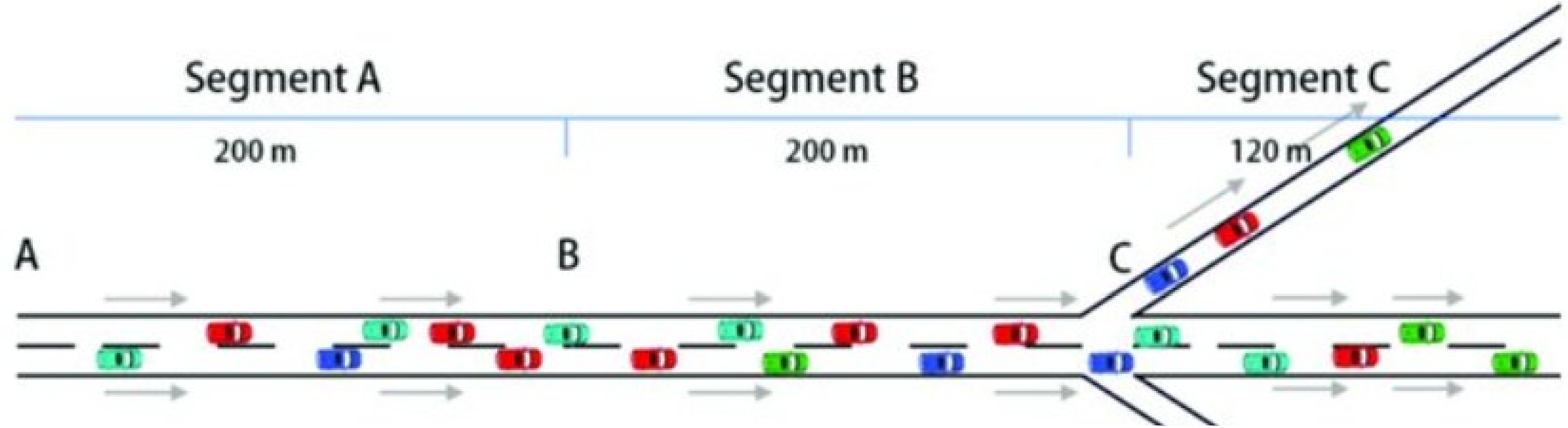
80 years anniversary of
LWR in 2025

Paul Irving Richards
(unfortunately, no
photo),
(1923–1978

Www.github.com/unicfdlab/libacoustics https://github.com/unicfdlab/libSonicBoom

Vehicular flows (VF): problem statement

- A segment of road is considered in order to estimate parameters of vehicles flow on it:



https://www.mdpi.com/1660-4601/16/22/4373

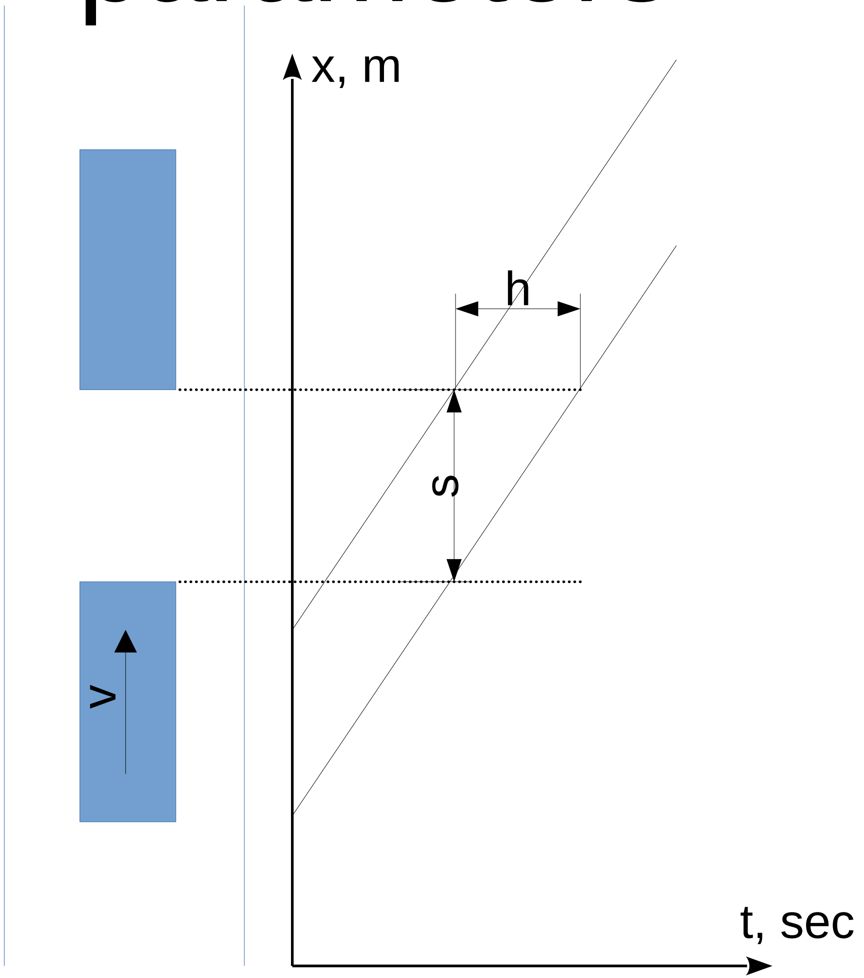
Traffic Simulation Analysis on Running Speed in a Connected Vehicles Environment

Individual vehicle parameters

- ***i*** – a label of a vehicle
- $\boldsymbol{x_i}$ - a position of a vehicle
- $\boldsymbol{l_i}$ – a path traveled by a vehicle during a time period
- $\boldsymbol{v_i}$ – an instant velocity of a vehicle

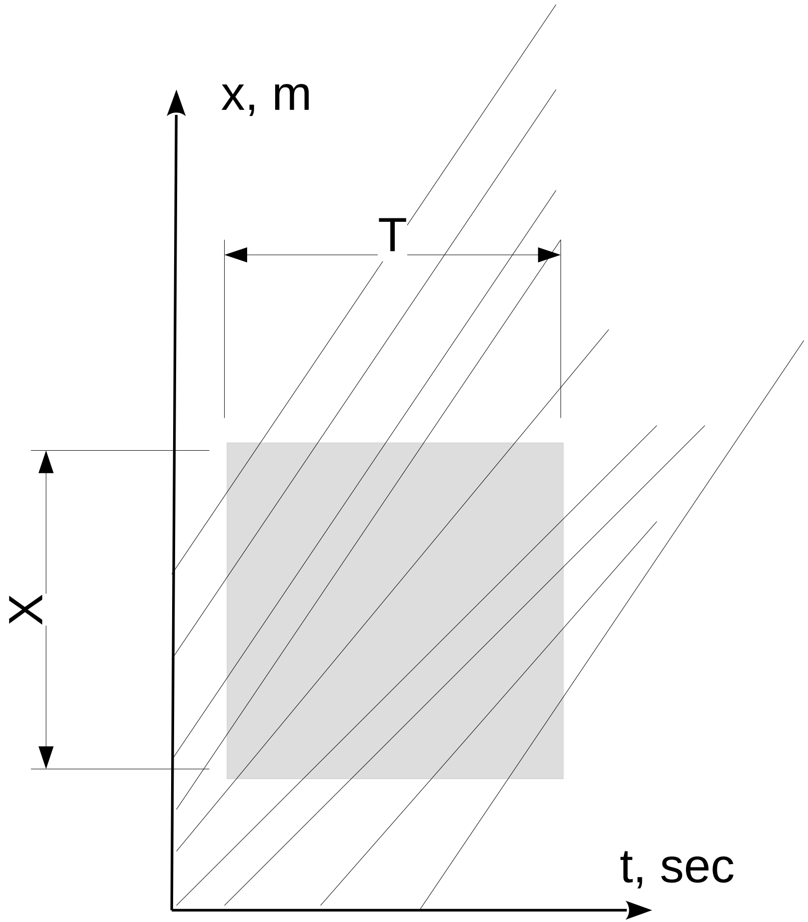
Microscopic VF parameters

- Basic parameters are:
  - headway, ***h = s / v***
  - space headway, ***s***
  - velocity, ***v***
  - pace ***p = 1 / v***
- All vehicles move in one direction
- See [22,26,27]



Macroscopic VF parameters [26, 28]

- ***N*** – number of vehicles
- Velocity ***u*** is an average for all vehicles occupying a region
- Flow ***q : N / T, 1/<s>***
- Concentration ***k : N / X, 1/<h>***
- ***q = k · u***
- Discrete / continuous



Physical meaning of k, q, u

- There is a good physical interpretation in [27]
  - ***q*** is a total distance of all vehicles during time T on the road segment X normalized by TX
  - ***k*** is the total time spent by vehicles during time T on the road segment X normalized by TX
  - ***u*** is a total distance of all vehicles during time T on the road segment X normalized by T
- Continuous

How *k* and *q* are inter-related?

- Three forms come from the conservation of vehicles principle [29]:
  - Integral $$\frac{\partial}{\partial t}\int_{x_1}^{x_2} k(x,t)dx = q(x_1,t) - q(x_2,t),$$
  - Second integral $$\int_{\mathcal{C}} -kdx + qdt = 0.$$
  - Differential $$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0.$$

Integral vs Differential forms

- Integral form is the essence of Godunov’s FVM method
- The integral form is applicable to continuous and discrete functions
- The differential form is applicable only between shocks

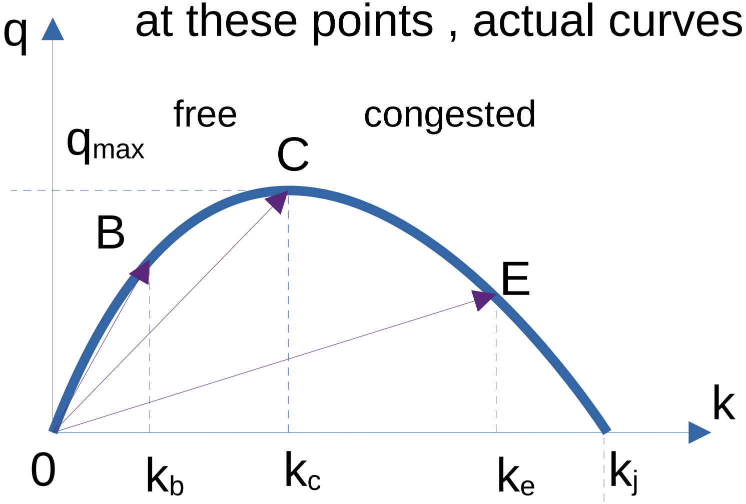
Density conservation equation

- Links changes of density and flow of vehicles
- No sources / sinks (no vehicles are destroyed or created)
- Predicts shocks evolution and phase transitions (due to shocks)
- Can be used to state the Riemann problem

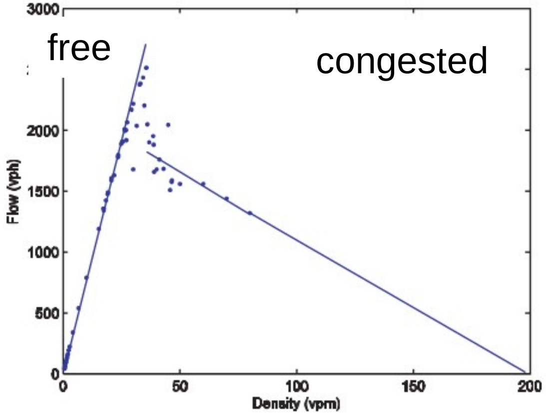
$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0.$$

Fundamental diagram [21, 25]

- Specifies F(k,q,u) = 0 for a stationary flow, slopes of 0B, 0E, 0C give space mean speeds at these points , actual curves are different (see[25])

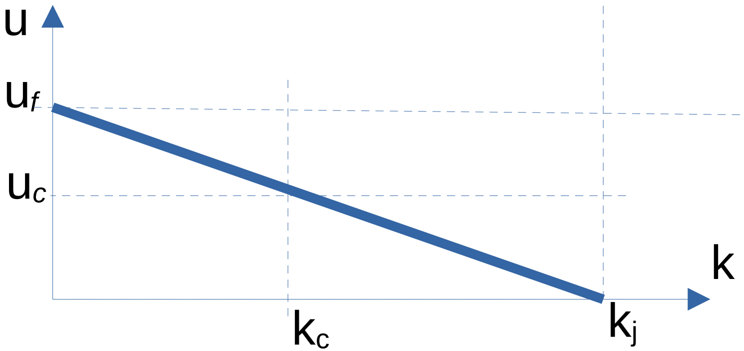


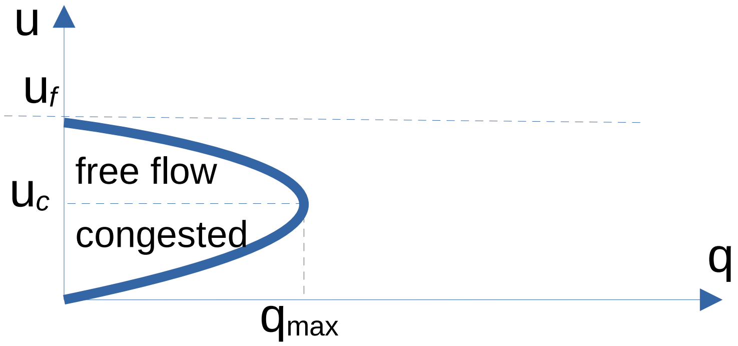
j – jammed
f – free
C – capacity/critical
congestion – deceleration,
jam – total freeze of a traffic



(a)

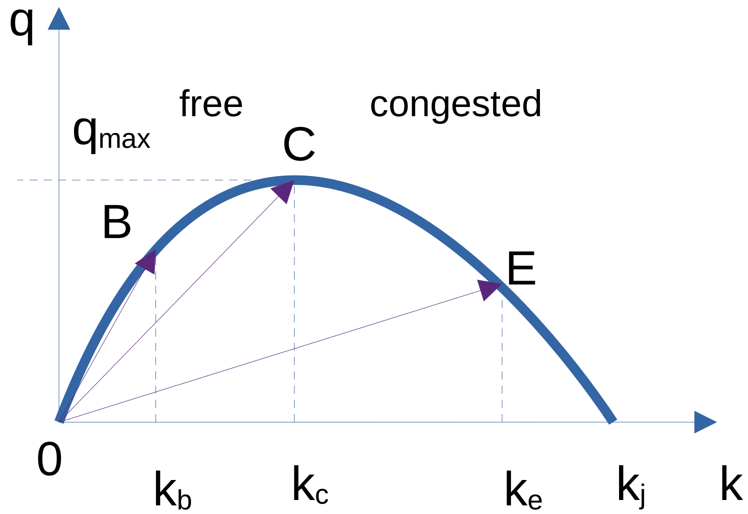
Real k-q diagram from [25]





Phase transitions

- Main vehicle flow phases [20, 23, 24], older:
  - Free traffic flow, dq/dk > 0
  - Congested traffic flow, dq/dk < 0
- Three-phase model [23, 24], modern:
  - congested traffic includes 2 sub-regions:
    - wide moving jams;
    - synchronized flow.
- Phase transitions can be treated as first order (q acts like Gibbs free energy if its derivative is discontinuous at k_c) with some assumptions

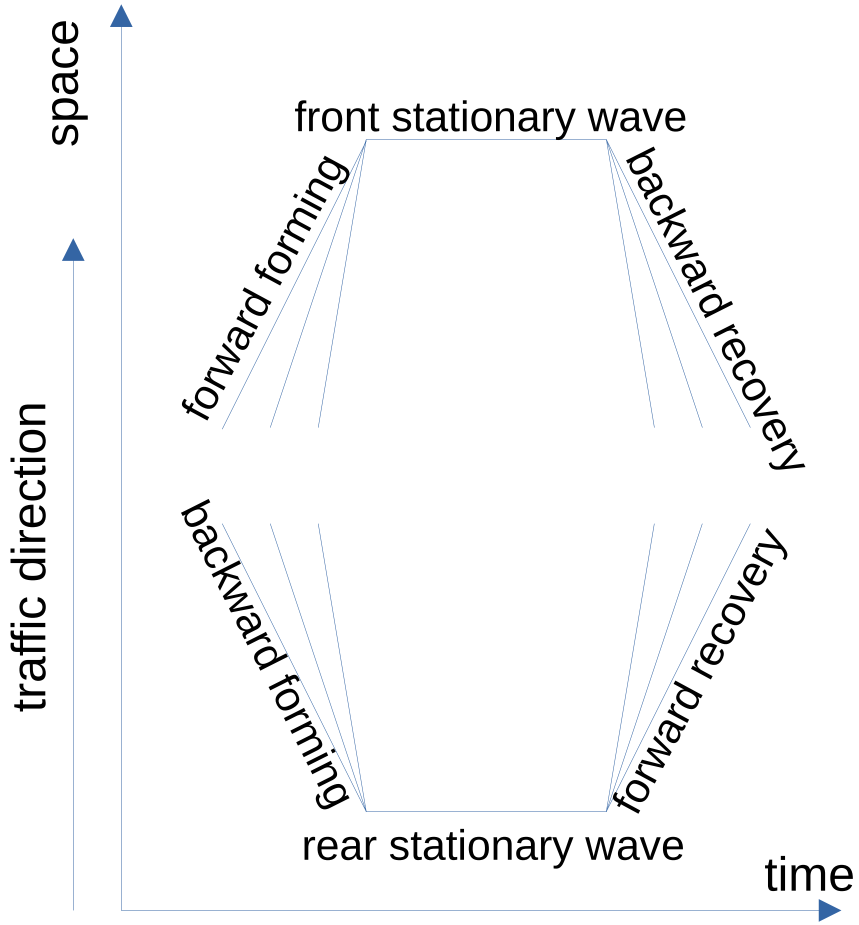


How phase transitions evolve

- Congestions happen due to: a) bottlenecks and b) self-organization reasons (stop-and-go waves)
- They travel up- or downstream depending on a situation
- From the Queuing theory (QT): when $\lambda > \mu$
- Shock propagation obeys R-H conditions [29]
- $u_s = [q] / [k]$

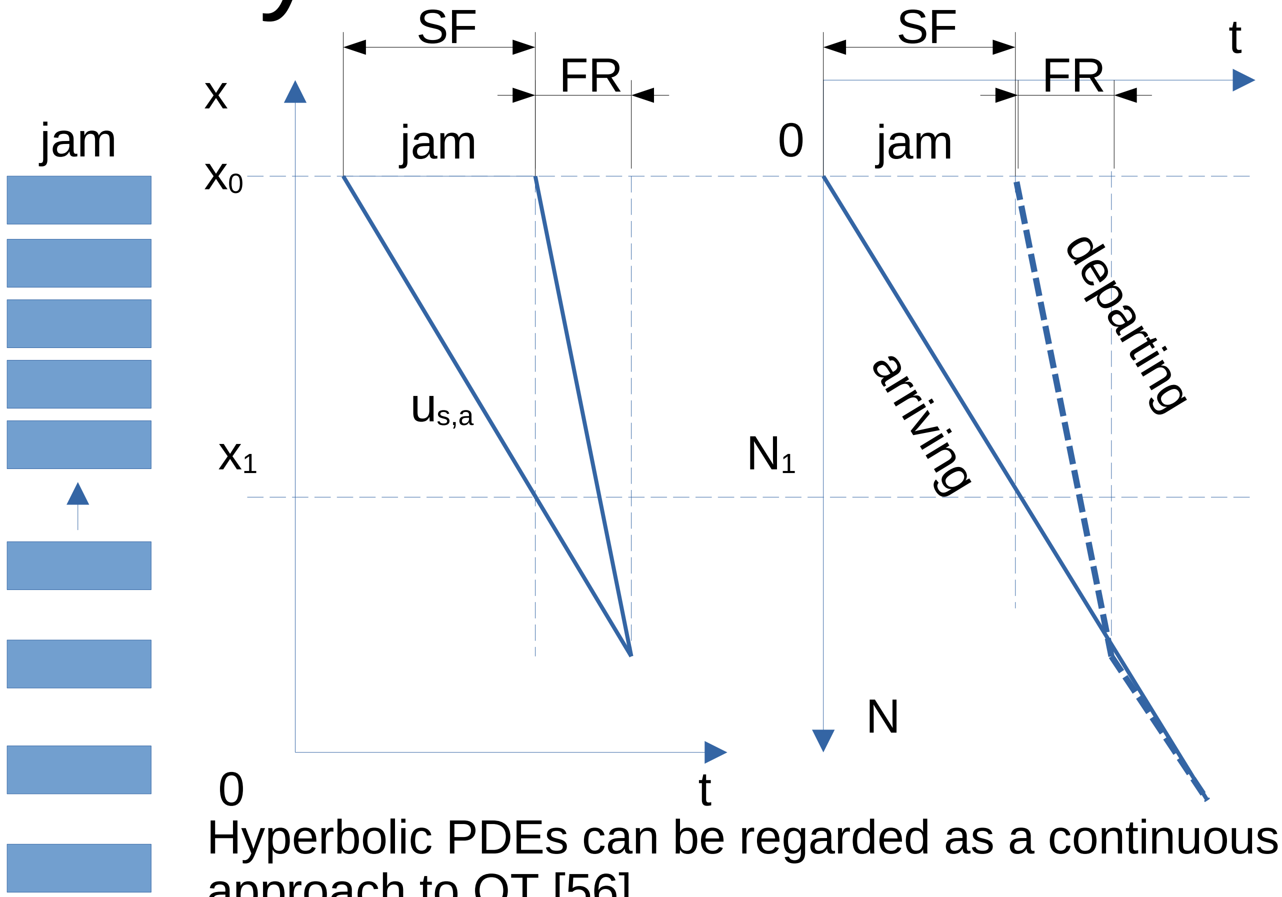
Shock waves types [4]

- front: an obstacle downstream
- rear: a standing wave upstream
- backward: shock propagates upstream
- forward: shock propagates downstream



Queuing theory vs Shock wave analysis

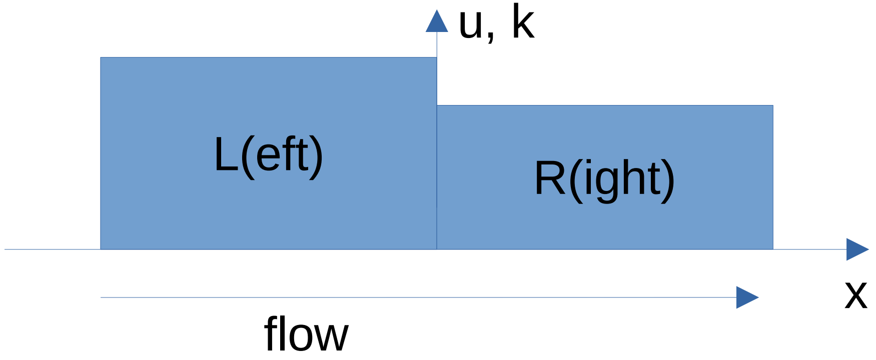
- λ – arrival rate (const)
- μ – service rate (var)
- k_a , k_d – the density of arriving & departing traffic
- $u_{s,a}$ – $-\lambda /(k_j - k_a)$
- $u_{s,d}$ – $\mu /(k_j - k_d)$
- N – cumulative vehicles count, $(x_0 - x)/k_j$

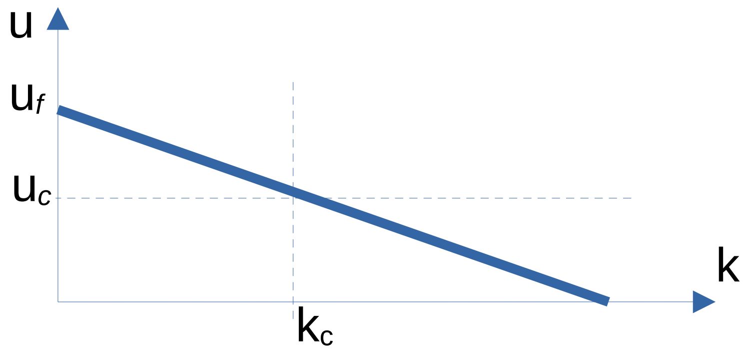


Hyperbolic PDEs can be regarded as a continuous approach to QT [56]

LWR Riemann problem

1) Both states are undercritical → expansion/shock
2) Left is critical, right is undercritical → expansion
3) Left is undercritical, right is critical → shock
4) Both states are critical (overcritical) → expansion/shock





Self-similarity and 1/f noise

- The solution of a LWR Riemann problem obeys self-similarity
- Also vehicular traffic shows self-similarity in a statistical sense [37, 38, 39]:
  - Hurst parameter $1/2 < H < 1$
- It was proved that vehicular traffic noise (arrival rate) obeys 1/f noise [5]

Other modelling approaches

- Queuing modeling (discussed above in brief)
- Kinetic theory’s Boltzmann equation for vehicular transport (Prigogine equation)
- Higher – order models (with fluid-like momentum equations)

Prigogine equation [29,32,28]

- [1]: “The Boltzmann-like model filled a lot of gaps in traffic flow modeling”: 1) it describes several lanes and 2) accounts for very light traffic
- [29]: “1. The validity of the slowing-down term is doubtfull …. 2. The absence of a derivation of the relaxation term from first principles...”

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = -\frac{f - f_0}{T} + c(\overline{v} - v)(1 - P) f.$$

Kurganov method for traffic [55]

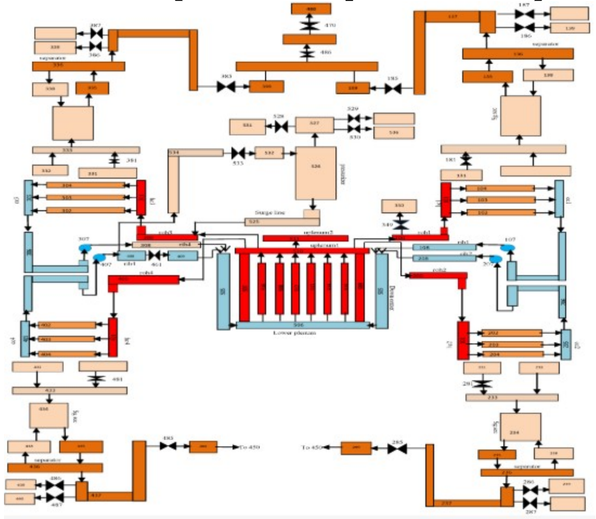
- This is actually a second-order model:
  - with LWR equation for free-flow
  - LWR-momentum equation for congested flow
- The method was originally developed for 1D gas dynamic flows (see the Appendix)

Gas dynamic flows

- Nuclear safety (RELAP5 mod3 code)
- Material balances in aircrafts
- Production – consumption networks (Gazprom)

https://www.upstreamonline.com/field-development/gazprom-finally-connects-rosneft-bp-operated-gas-field-to-pipeline-network/2-1-917232

https://doi.org/10.3390/jne2040035



1D gas dynamics equations[51]

Mass conservation for an area-averaged properties <X>:

$$\frac{\partial}{\partial t}\left(<\rho> A\right) + \frac{\partial}{\partial x}\left(<\rho u> A\right) = 0\,.$$

Momentum balance for an area-averaged properties <X>:

$$\frac{\partial}{\partial t}\left(<\rho u> A\right) + \frac{\partial}{\partial x}\left(<\rho u^2> A\right) = -\frac{\partial}{\partial x}(<p> A) + p_w \frac{dA}{dx}\,.$$

Energy balance for an area-averaged properties <X>:

$$\frac{\partial}{\partial t}\left(<\rho e_0> A\right) + \frac{\partial}{\partial x}\left(<\rho e_0 u> A\right) = -\frac{\partial}{\partial x}(<pu> A)\,.$$

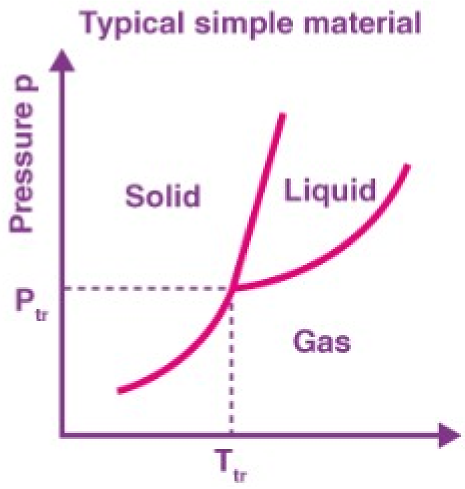
These set of equations can be derived:
- phenomenologically (by considering balances of mass, momentum and energy)
- or from the Boltzmann equation using Chapman–Enskog theory

Closure relations for gas

- Equation of state $F(\rho, p, T) = 0$
- Constitutive relations: H=H(p,T), S=S(p,T), ...
- Gibbs free energy: G = H – TS
- Maxwell relations

Phase change

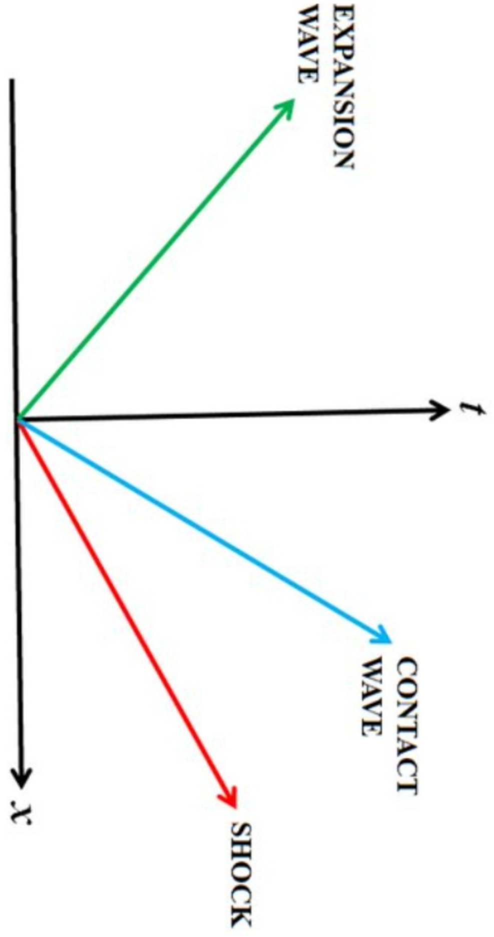
- Is a situation when properties change abruptly (incl EoS)
- With some simplifications:
  - 1st order: G is continuous, S and V are discontinuous, latent heat
  - 2nd order: G is continuous, S and V are discontinuous



https://byjus.com/chemistry/phase-diagram-of-water/

Riemann problem

- 3 waves (instead of 1 for LWR)
- R-H for shocks, continuity for primitive variables on a contact and isentropic relation fo expansion

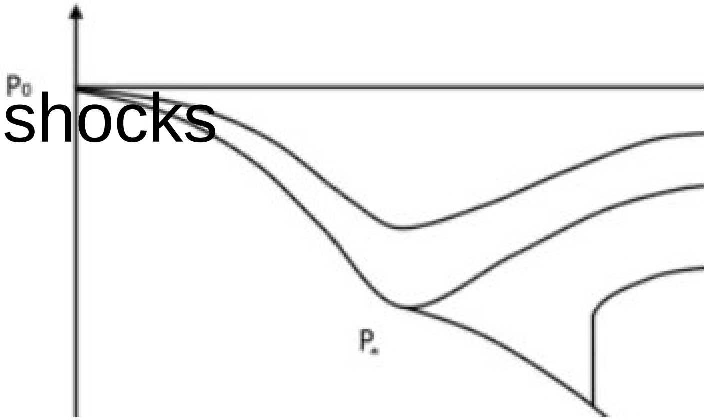


https://doi.org/10.13140/RG.2.2.22682.67526

Normal shock in a converging-diverging nozzle [53]

- Pressure is converted into high-speed flow
- High-speed flow:
  - creates a shock (due to converging characteristics)
  - increases entropy by converting mechanical energy into the internal
- R-H + thermodynamics → positions of shocks
- Energy balance plays the role of FD

$$[\![\rho V_n]\!] = 0\,,$$
$$[\![\rho V_n^2 + p]\!] = 0\,,$$
$$[\![\boldsymbol{V}_t]\!] = 0\,,$$
$$[\![h + V_n^2/2]\!] = 0\,.$$



Flows in a two-phase region

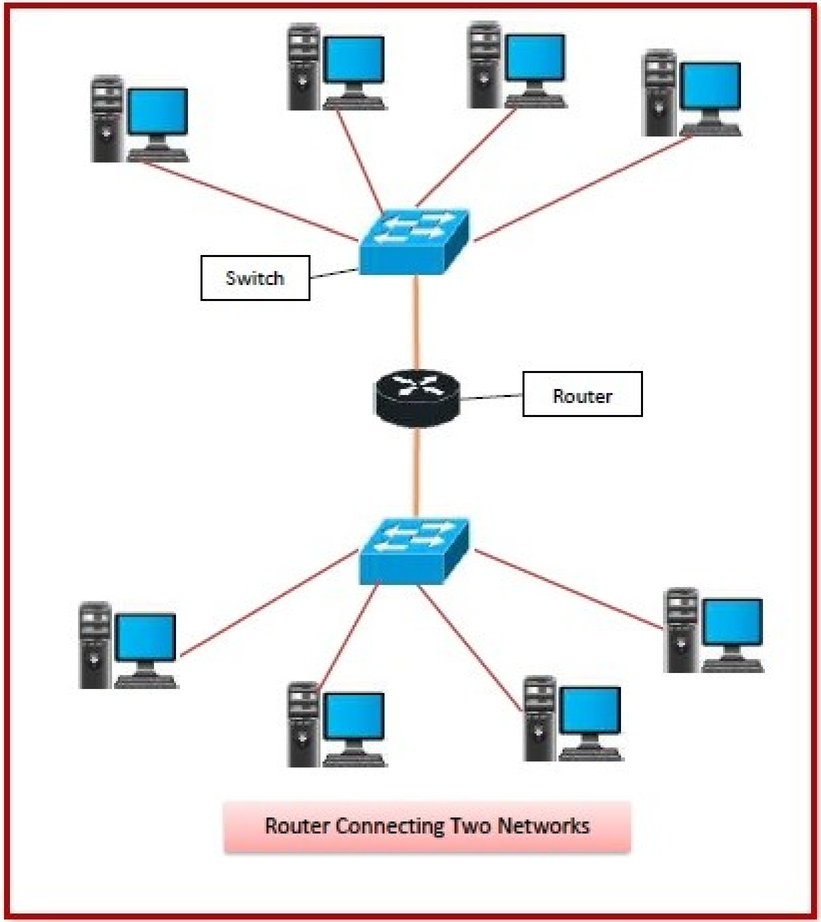
- Thermodynamic relations → high-level estimations of wave speeds → simplified models with artificial compressibility → shockwave patterns
- Simpler and adequate solutions in compressible 2-phase regions for the same set of equations [60]

Granular (two-phase) flows and data flows

- Several researchers note similarities between granular (two-phase) flows in pipes and discrete packets flows in computer networks [47, 61]:
  - burstiness;
  - self-organization;
  - shocks / congestions;
  - Etc
- There is a special conference “Traffic and Granular flows”, since 1995

Computer networks flows

- Carrier: electromagnetic waves
- Media: copper wires, optical cables, wireless, AC circuits
- Propagation speed: ~ 10^8 m/s
- The notion of “data fragments” is not tangible



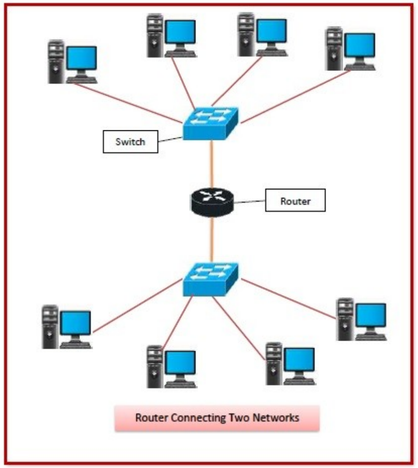
https://www.tutorialspoint.com/

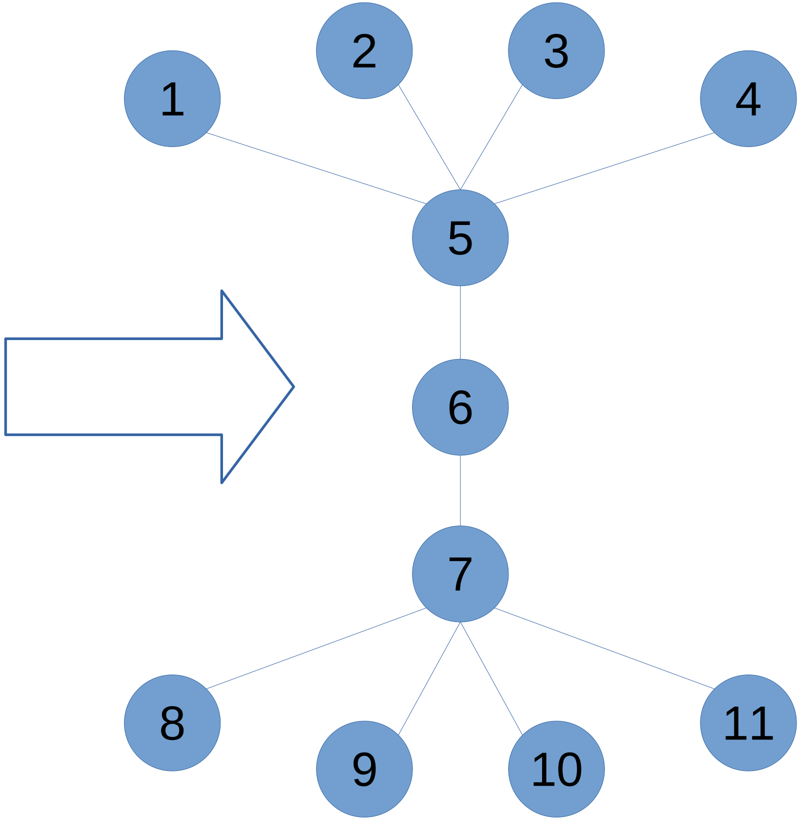
Computer networks modeling

- Network flows theory (NFT): only steady-state picture
- Queuing theory / discrete event simulation (QT/DES): accurate, but resource-demanding
- Continuous (Fluid) models: less accurate, but also less resource-demanding

Network flows theory

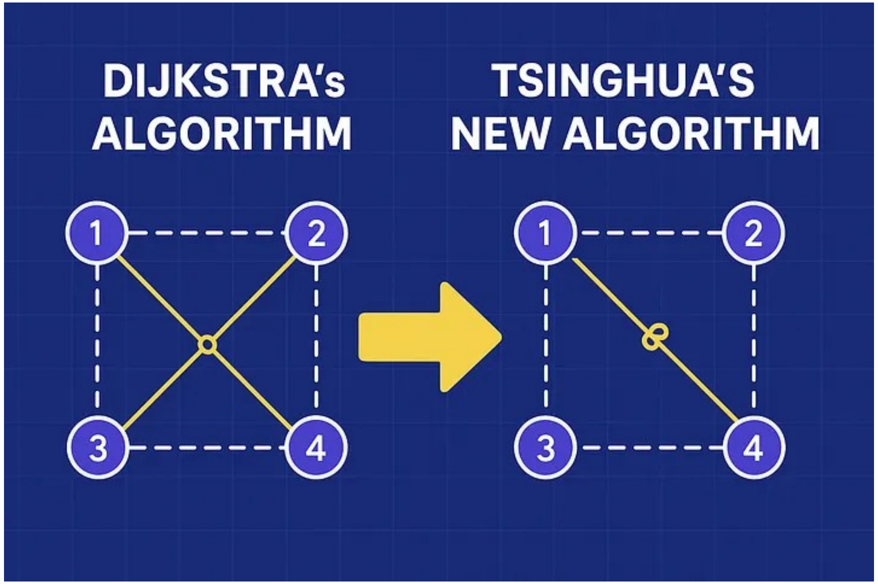
- Considers topology of a network: nodes + edges
- Usually answers questions of:
  - shortest path
  - maximum flow
  - minimum cost
  - etc





Recent Tsinghua discovery [57]

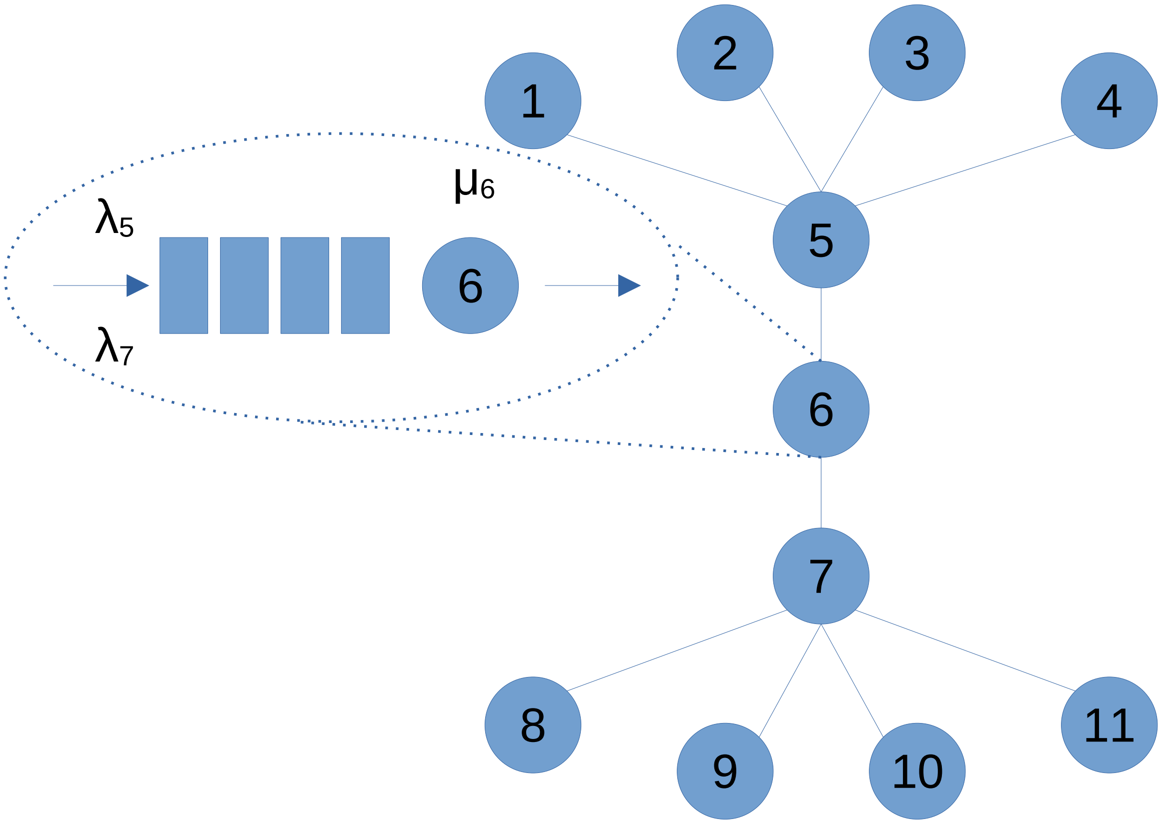
- An algorithm faster than Dijkstra (2025):
  - Original Dijkstra's algorithm for shortest path problem complexity: O(m + n log n)
  - New Tsinghua algorithm complexity: $O(m \cdot \log^{(2/3)} n)$



https://medium.com/@vverma4313/tsinghua-university-breaks-a-65-year-limit-a-faster-alternative-to-dijkstras-algorithm-e2f42a608369

MC/QT[3]/DES[34] simulations

- Considers transient processes in computer networks:
  - each node is characterized as a queue(s)
  - prediction of delays and services availability / capability
  - simplified simulation of typical situation

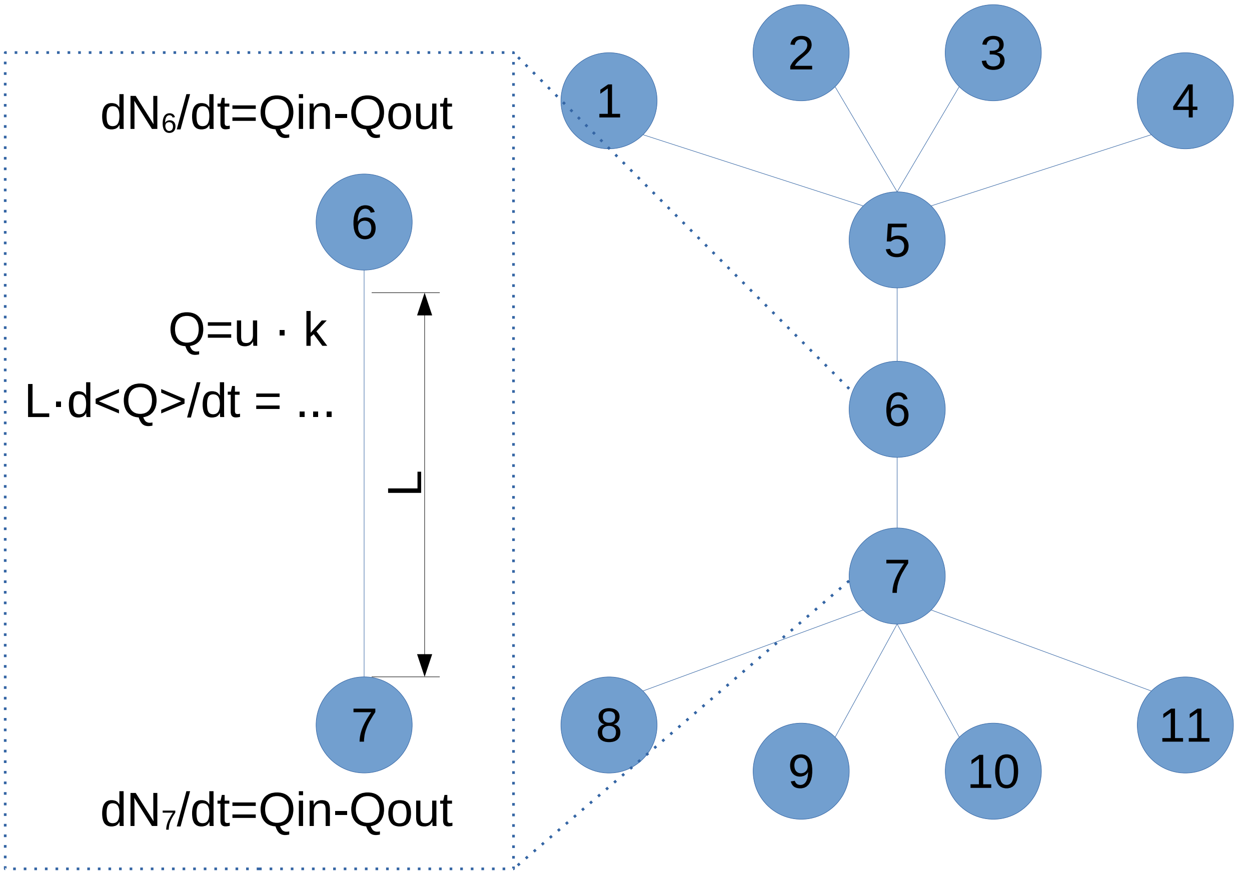


Limitations of QT/DES

- Transient behavior is possible only for a microscopic approach → expensive [6]
- Only discrete time instants are considered
- Questions of inertial effects (what about hysteresis?)

The main idea of fluid models

- Consider data flow (stream of packets) as a continuous medium, introducing:
  - A density function, an equation of state, propagation equations + balance equations.



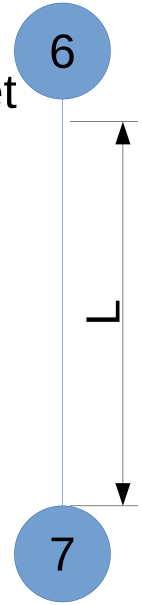
Fluid models for data networks taxonomy

- Direct simulation (linear wave equation)
- Artificial compressibility (non-linear transport)
- Incompressible fluid (stochastic differential equations) models
- Boltzmann equation (kinetic theory)

Direct simulation

- A simple linear equation is considered between nodes (routers):
  - a wave equation for a bit propagation, e.g. [33]
  - a convection equation for a packet transportation
- A conservation equations is considered inside nodes (routers), since each router has at least 1 queue
- $t_P << t_L$
- Balance equations inside nodes
- Almost useless, but establishes a base for next two approximations

$t_P = n_b \cdot L_b / c$
$t_L = L / c$
$c = 2 \cdot 10^8$
n_b – bits per packet
L_b = 2 m (100 Mbit/s)



Artificial compressibility

- Seminal work (derivation from first principles): [6]
- Phenomenological derivation [7,8]
- Recent works [9,10] using [7,8]
- Second order models [35]

Derivation of a transport equation[6]

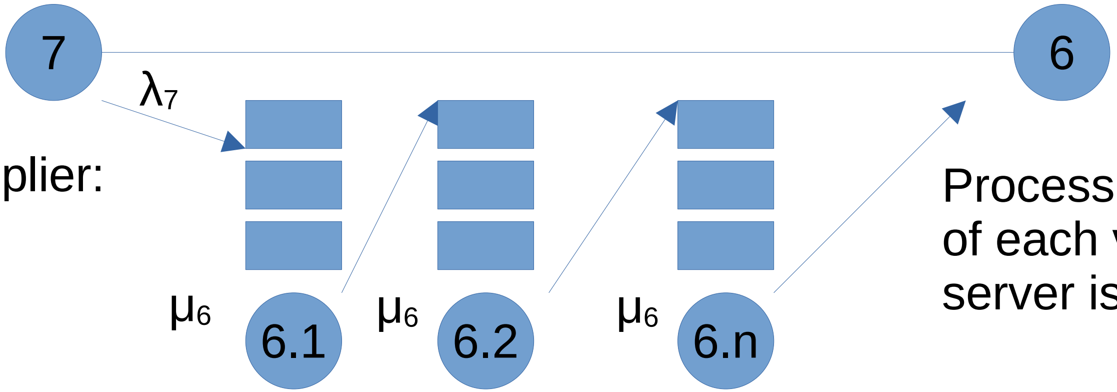
- The model is derived from first principles
- Authors directly relate DES with Monte-Carlo/Lagrangian methods and flow models with gas dynamics
- Convection equation in integral form is derived directly from conservation of data in a queue:

$$\frac{\partial k}{\partial t}+\frac{\partial q}{\partial x}=0 \quad q=min(\mu, W/T) \quad W=\int k\,dx$$

Fictitious spatial axis

- The conservation equation is derived for a sequence of many servers (suppliers in [6])
- But by introducing along a link a chain of “virtual suppliers” with characteristics equal to original server, we can describe computer networks:

Each virtual supplier:
$dk/dt = \mu_{in} - \mu_{out}$



Processing time of each virtual server is T/n

Incompressible fluid approximation

- If a link is too short to accommodate a packet as a wave, then a flow must be considered as incompressible.
- According to [14-19], if number of packets is large enough, their individual stochastic behavior degenerates into predictable macroscopic relations formulated using ODE.
- However, no connections with temperature, pressure or other physical quantities are mentioned.
- “we allow the packet size to be arbitrarily small. We refer to this as the fluid model.” [14]

Fluid approximation for TCP

- This approximation is frequently used for TCP data flows (for example, between nodes 6 and 7)
- W – congestion window (number of bytes to send with acknowledgment);
- T – round trip time (source of W)
- W(t)·P(t) – sink of W

$$\frac{dW}{dt} = \frac{1}{T} - \frac{1}{2} W(t) P(t),$$

Important applications of SDN and fluid models

- The authors of [16] emphasize:
  - the important role of simulations (DES and fluid) for traffic shaping using SDN solutions;
  - application of SDN and wireless networks and their importance for IoT and industry 4.0.

Boltzmann equation

- In the early 2000s, I. Antoniou, V.V. Ivanov, Yu.L. Kalinovsky proposed a kinetic model approach [11-13] and derived a Boltzmann equation for data flows
- Their approach was inspired by prof. Prigozhin who used it for vehicular networks earlier.
- Although there was no continuation for this model, it sill looks as exceptionally promising, see [1] for example.

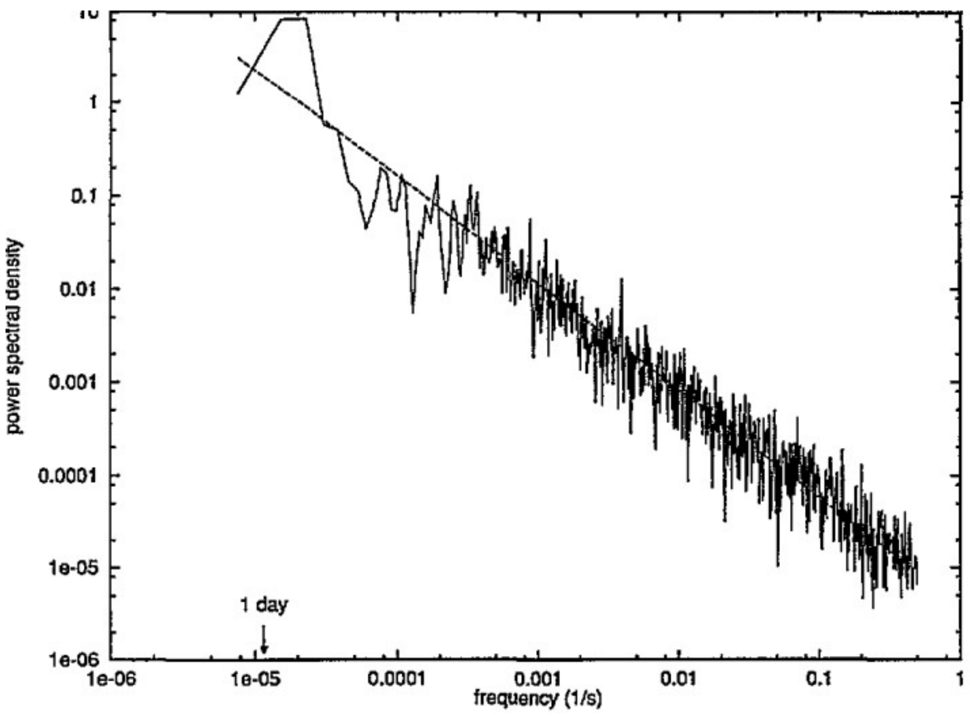
$$\frac{\partial f(x,v,t)}{\partial t} + v\,\frac{\partial f(x,v,t)}{\partial v} = -\frac{f(x,v,t) - f_0(x,v,t)}{T} + c(\bar{v}-v)(1-P)f(x,v,t)\,.$$

Self-similarity of data flows in computer networks

- Many authors ([36, 37] , for instance) report statistical self-similarity of Ethernet data traffic
- Why it matters:
  - We can extrapolate statistical data from small time ranges to longer
  - Vehicular traffic manifests the same statistical self-similarity [38, 39], therefore we’ll encounter similar behavior of models which are based on similar statistical assumptions

1/f noise

- In 1995, prof I. Csabai discovered the 1/f noise in computer networks[40]
- Although other authors do not confirm it for all operational regimes, it was proved for selected cases [41]
- The discovery had urged I. Csabai to study similarities between vehicular and computer networks traffic [42]



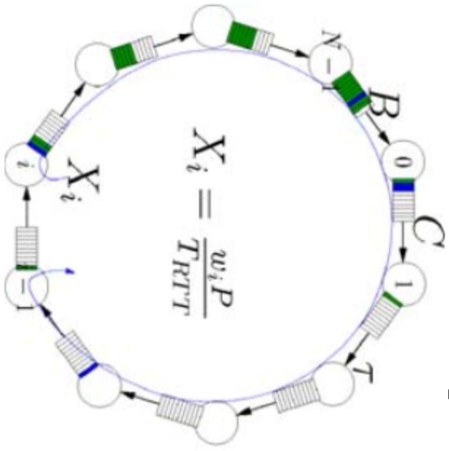
Phase transitions

- Depending on μ and λ values [43,46] a queuing system might be in a:
  - free state, when $\lambda < \mu$ and delays are limited;
  - transitional state, when $\lambda = \mu$ when all buffers are full;
  - congested state, when $\lambda > \mu$ when buffers (and delays) grow infinitely.
- For TCP-type protocols the transition is not so sharp because a congestion control works, but a distinction between states still present [44,45]
- 1/f law corresponds to moderate congestion (intermediate regime)[45]

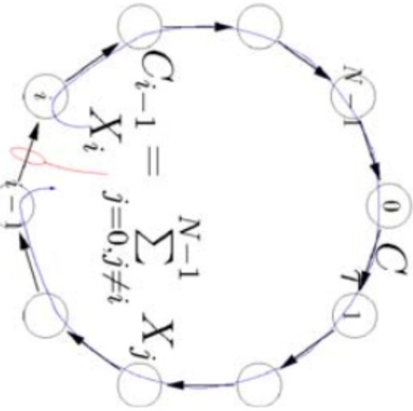
Tretyakov et al mention that transition is 2^{nd} order, however it is not clear why he decided this

Congestion waves propagation

- The similarities between vehicular, granular and data flows are noted [47].
- The authors[47] studied congestion waves propagation using Berkeley Network Simulator [48, 49] DES and a simplified fluid model.
- The authors have shown correctness of the simplified fluid model for the problem of congestion wave propagation.
- A formula for the congestion wave speed was derived.



Continuous model [47]



Discrete model [47]

Possible solution approaches

- The considered approaches should yield a hierarchy of continuous models for data flows: from Boltzmann equation as the most general representation to fluid models as the most empirical approach.
- Since all these problems have been well-studied in gas dynamics, it is expected that present methods (RK, LBM, KNP, QGD) are also applicable for corresponding problems.

CONCLUSIONS

Similiarities I

- Dynamics of networks of all considered types might be described:
  - either using Boltzmann (Boltzmann-type) equations
  - or using hyperbolic PDEs
- All PDE-based models share it least conservation of 1 property (density / concentration)
- Similar methods can be used for numerical analysis or solution of the corresponding PDEs

Similarities II

- Dynamics of all systems (gasdynamic, vehicular and computer) might demonstrate shock waves and rarefaction waves, transient and stationary shocks
- Phase transitions are also possible in all three type of networks, although their nature is different
- Phases can have metastable states [20, 23, 24]
- Hyperbolic PDEs might work as an alternative description of queues
- Statistics of both vehicular and data flows can demonstrate self-similarity and 1/f noise

Differences

- Vehicles never collide, but still interact
- Dense gas molecules collide, rarefied gas molecules event don’t collide
- Data packets follow each other, but don’t collide and don’t interact

Motion types

- Motion of vehicles is purely translational and 1D (2D at max [58])
- Monotatomic perfect gas is 3D and has translational degrees of freedom, other gases exhibit more complicated motion
- Packets flows in data networks can be considered as also 1D and translational

Macroscopic properties

- Vehicular networks
  - Density, pcs / m
  - Velocity, m / s
  - Flow, pcs / s
- Gas dynamics networks
  - Density, Temperature (which is a specific kinetic energy) and Pressure (which is a force), velocity (m/s) + thermodynamic functions
- Computer networks
  - Density, pcs / queue
  - Inverse of RTT, 1 / s

Shock wave profile

- Gas dynamics: steepens for high-speed flow [52] thanks to energy equation
- Vehicles flow: steepens for q(k) with curvature [50]
- Packets flow: q(k) is usually linear

Equation of state

- Vehicular networks use “FD” which play simultaneously roles of constitutive relations, equation of state, properties of an environment (a road segment) and equation of state
- In gas dynamics networks all these functions are separated between different equations
- In computer networks these questions are understudied, but situation is closer to vehicular networks

Entropy meaning

- Vehicular networks: might be defined using Shannon’s princinple, headway or velocity [59], but still there is no final definition.
- Gas dynamic networks: there is well-developed theory for estimation of entropy.
- Computer networks: probably, Shannon’s approach, but also the issue looks like as under-developed.

Phase changes

- For VN, they can be classified as 1st order if q is considered as Gibbs free energy
- For GN, usually, 1st order transitions take place
- For CN, some authors attribute (Tretyakov et all [44]) transitions as 2nd order, but the reason for the classification is not clear

Challenges I

- Packets loss modeling [15,17]
- Models [14-19] are for TCP, for other protocols new models are needed
- In SDN we have virtual paths in addition to physical links, therefore, the continuous approach requires adaptation to virtual environment

Challenges II

- Curvilinear geometries of roads influence density distributions and must be accounted, as well as junctions and inter-lane interactions.
- Phase space classification for VN and CN as not finally molded
- Lack of a reliable procedure to build a FD for physical and virtual computer networks.

Conclusions

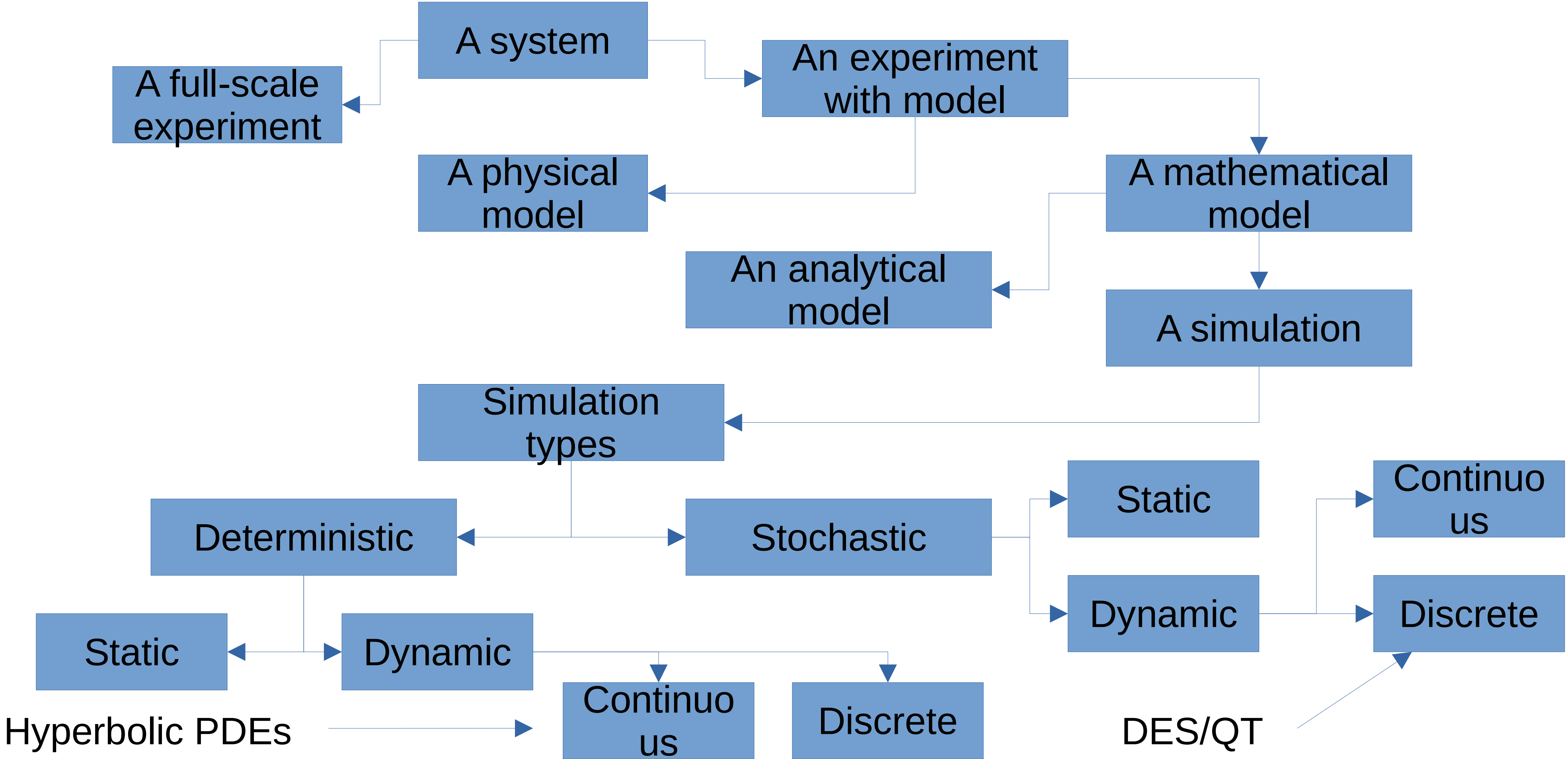
- Whether these analogies between gasdynamics-, vehicular- and computer- networks are justified or not, the amount of transferred data will continue growing, and the demand for tools providing prediction and management of data flows will increase too.
- Thermodynamics of vehicular and computer is underdeveloped, thus hindering their practical usage.
- Practical benefits for SDN: an assessment of a network throughput dynamics due to demanded changes.
- Although domains of computer & vehicular networks might seem as least familiar for specialists in gasdynamics, they have many traits

APPENDIX

Programming and modelling

- ASM: micro-processor instructions
- C: systems programming
- Fortran: numerical computations
- SIMULA: Descrete Event Simulations (QT/MC)
  - OOP came from modelling
- C++: DES + systems programming
  - C + SIMULA
  - OOP concept (simulation) and fast (due to low level operations)

Models tree

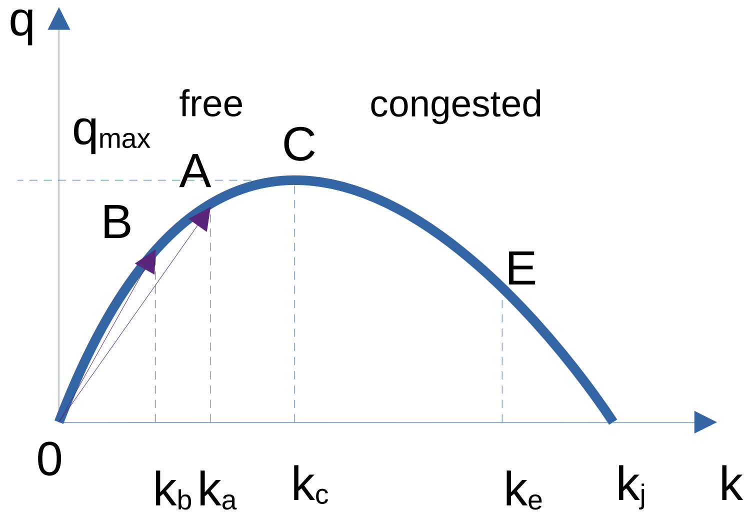

A system
A full-scale experiment
An experiment with model
A physical model
A mathematical model
An analytical model
A simulation
Simulation types
Deterministic
Stochastic
Static
Dynamic
Continuous
Discrete
Static
Dynamic
Continuous
Discrete
Hyperbolic PDEs
DES/QT

SDN/DES

- SDN: virtualization, but also a simulation (imitation of networks)
- Example of DES for networks: Berkley NS [48,49], C++
- As a simulator, SDN relies on DES/QT
- Hence, the choice of OOP for OpenSDN and other similar technologies

Vehicular shock wave definition

- By equating flxues on two sides of a shock $q'_b = k_b(u_b - u_s) = q'_a(u_a - u_s)$ we get the speed of the shock [4,21,22]: $u_s = (q_b - q_a) / (k_b - k_a)$
- This is known as Rankine-Hugoniot conditions [29]
- If $k_b \rightarrow k_a$, then $u_s \rightarrow c$ [29]
- Other derivations are in [29]
- Qualitative explanation [50]



Solution methods of hyperbolic PDEs

- PISO/SIMPLE (projection-type)
- AUSM [65]
- Characteristics-based (HLLC, etc)
- TENO [64]
- KNP [62]
- QGD [63]
- Hybrid PIMPLE/KNP (for viscous subsonic flows)

QGD

- Established by: B.N. Chetverushkin, T.G. Elizarova and Yu. V. Sheretov
- No flux limiters

Prof. Boris N. Chetverushkin

Prof. Tatiana G. Elizarova

Prof. Yurii V. Sheretov

- No Riemann problem solution is needed
- Straightforward regularization procedure
- Successful application for many problems (including subsonic and viscous flows)
- OpenFOAM implementation[63]: https://github.com/unicfdlab/libAcoustics

Kurganov – Noelle – Petrova method

- The original paper [62], 2001
- Considers spatial-temporal average of gas-dynamic PDE + 2 waves (compression and expansion)
- Golden standard for many codes
- Applicable to many types of flows

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Limitations of FD

- From [025] there are evidences that q(k) is not concave → higher order models?
- Is it an equation of state or a constitutive relation?
- How can we select its shape from observations?

A connection with artificial compressibility

- Distribution of a queue properties along a link between nodes leads to a lower speed of data propagation in space
- Lower propagation of speed corresponds to larger (additional) compressibility

Recent uses of [7] and [8]

- The authors of [9] postulate the transport equation with a simple FD (k-q) and fictitious spatial dimension [0, 1]
- The authors of [10] employed the approach to study congestion and waves in SD-WAN configurations

Second order modeling [35]

- A Mathematical Model of Network Communication by Donald A. Drew
- An attempt to derive a momentum equation for packets
- The author is known for his works in traffic theory pertaining to bridging statistical physics notions with vehicular flow
- Unfortunately, no significant continuation, but can show path for further work

$$\partial_t v + v \cdot \nabla v = -\nabla \omega \qquad \frac{\partial \rho}{\partial t} + \nabla \cdot \rho u = 0$$

Highway level of service vs quality of service

- Level of service (LOS) is a qualitative measure used to relate the quality of motor vehicle traffic service
- Quality of service (QoS) is the description or measurement of the overall performance of a service

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